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RESEARCH ARTICLE

RESEARCH ON WEATHER DERIVATIVES PRICING--THE CASE OF SHANGHAI MUNICIPALITY

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ABSTRACT

Weather derivatives pricing is one of the central issues in the study of this type of financial product, and there is no uniform methodology. To price the temperature option with Shanghai temperature as the underlying and explore how to improve the accuracy of option pricing, firstly, the time-varying O-U model is combined with Monte Carlo simulation to obtain the Shanghai-based temperature option pricing, and then Shanghai and its neighboring Dongtai, Quxian, and Dinghai are selected to constitute an option portfolio and priced using the same method. The results are obtained: 1) the predicted price of each unit of Shanghai temperature option is 1732.33 yuan, and the actual price is 1557.84 yuan, with a relative error of 9.1%; 2) the predicted price of each unit of option portfolio is 1598.12 yuan, and the actual price is 1500.72 yuan, with a relative error of 6.5%; and 3) the same pricing steps are repeated several times, with a very robust relative error. It can be seen that the pricing method has stability and higher prediction accuracy and can be used in practice. At the same time, pricing after selecting multiple cities to form a weather derivative portfolio has higher accuracy i.e. less risk than pricing only for a single city.

KEYWORDS

weather derivatives, time-varying O-U model, Monte Carlo method, European call option, risk hedging

1. INTRODUCTION

With its vast territory and complex and variable climate, China is one of the countries in the world that suffer the most from weather disasters. According to China Emergency Management News, in May 2023, weather disasters affected 919.6 thousand hectares of crops; direct economic losses amounted to RMB 8.45 billion. Particularly, China is a large agricultural country, and according to the official website of the State Council, the agricultural output value accounted for 16.05% of GDP in 2021. As agriculture is significantly affected by weather, unpredictable and severe weather often causes economic losses to practitioners, and weather derivatives can provide an effective risk-hedging tool for agriculture and its related industries to reduce economic losses. Meanwhile, the introduction of weather derivatives as a financial product will enhance the completeness of China's financial market and promote the diversification and internationalization of the financial market. On May 21, 2021, Daxin renewed its strategic cooperation agreement with the National Meteorological Center, focusing on the compilation of indexes for forecasting the yield of major crops such as accumulated temperature and precipitation, as well as the research and development of other temperature indexes and related derivatives on June 10, 2021. The National Meteorological Information Center and Zhengzhou Commodity Exchange signed a strategic cooperation framework agreement to jointly carry out feasibility studies, variety design, market opinion solicitation, and listing of weather derivatives such as weather index futures.

Among the many weather derivatives, temperature derivatives have the longest history of development and are the most thoroughly researched, so they are naturally the focus of our development. Such derivatives are benchmarked against a temperature index, so a forecast model on temperature needs to be obtained before pricing. In 1977, a researcher first proposed the O-U (Ornstein-Uhlenbeck) model, which is characterized by mean reversion, in his study of the interest rate problem (Vasicek, 1977). Since the change of temperature also conforms to the characteristics of mean reversion, a scholar in 1998 first used the O-U model for temperature prediction and established a stochastic differential equation describing the change of temperature (Dischel, 1998). After that, many scholars improved the O-U model to enhance the prediction accuracy of the model. A sinusoidal function could be used to fit the seasonal trend of temperature and the volatility of temperature could be considered as a segmented function that varies on a month-to-month basis (Alaton, 2002). If the volatility of temperature was accurate to the day it could improve the accuracy of the model prediction (Bhowan, 2003). After that, a researcher introduced the Fourier transform based on the daily volatility model and used multiple sine and cosine functions to describe seasonal temperature changes, and the model became the most popular temperature prediction model. Domestic research on the pricing of temperature derivatives started late (Benth and Saltyte, 2007). A scholar used the O-U model to model and forecast the temperature in Shanghai (Li et al., 2011). A group of researchers used the time series model, the O-U model based on monthly volatility, and the O-U model based on daily volatility to model and compare the forecasting effect of the temperature; and found that the O-U model based on daily volatility was the optimal one (Wang et al., 2015). The model in which the rate of return and volatility of temperature varies with time is called the time-varying O-U model. In addition to the O-U model, there are other methods for temperature prediction, such as neural networks and time series. In 2012, two researchers used BP neural network to predict temperature (Tu and Wang, 2012). In 2015, two scientist used a wavelet neural network to predict temperature (Wang and Gou, 2015). and in 2022, While some researcher used the ARMA model to predict temperature (Wang, 2022).

Based on the above background, this paper selects the time-varying O-U model as the theoretical basis and the Monte Carlo method as the

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implementation algorithm to study the pricing problem of temperature derivatives with Shanghai as an example, hoping to help the development of China's weather derivatives market. At the same time, considering the randomness of the Monte Carlo method, to improve the accuracy of pricing, after being inspired by the purchase of multiple stocks in the stock market to hedge investment risks (Li et al., 2022). Shanghai and its neighboring Dongtai, Quxian, and Dinghai are selected to constitute an options portfolio for pricing, comparing whether there is any difference in the accuracy of pricing between the selection of a single city and the selection of multiple cities.

2. METHOD

2.1 Modeling

The traditional O-U model treats both the rate of return and the rate of fluctuation of temperature as constants, and its stochastic differential equation is expressed as follows:

$$dT(t) = dS(t) - K(T(t) - S(t)) + \sigma dB(t) \tag{1}$$

Where:$T(t)$ denotes the actual temperature,$S(t)$ denotes the long-term and seasonal trend (also known as the deterministic trend) of temperature,Kdenotes the rate of return of temperature, σdenotes the volatility of temperature, and$B(t)$ denotes the standard Brownian motion. Where$K$,$\sigma$is a constant. Based on this model, we consider the rate of return and volatility as segmented functions varying monthly to obtain a time-varying O-U model:

$$dT(t) = dS(t) - K(m)(T(t) - S(t)) + \sigma(m)dB(t) \tag{2}$$

Considering that in practice, we use days as the basic unit, the above equation is obtained by discretizing it:

$$\Delta T(t) = \Delta S(t) - K(m)(T(t) - S(t)) + \sigma(m)\Delta B(t) \tag{3}$$

The resulting recursive equation for the temperature prediction is:

$$T(t+1) = T(t) + \Delta T(t) \tag{4}$$

To obtain a concrete expression of the time-varying O-U model, we need to estimate $S(t)$, $K(m)$ and$\sigma(m)$. Their estimation methods are described below.

2.2 Estimation of $S(t)$

Since there are both long-term and seasonal trends in temperature, a combination of linear and sinusoidal functions are considered for fitting:

$$S(t) = A + Bt + C\sin(\omega t + \phi) \tag{5}$$

For modeling convenience, the data for February 29th in a leap year is excluded, so that $\omega = 2\pi/365$ i.e., the period is 365 days. Expanding the above equation trigonometrically yields:

$$S(t) = A + Bt + C(\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi) \tag{6}$$

Let $d = C\cos\phi$, $e = C\sin\phi$, $t_1 = \sin(\omega t)$, $t_2 = \cos(\omega t)$, Equation (5) be equivalent to estimating the coefficients of the following linear regression equation:

$$S(t) = a + bt + dt_1 + et_2 \tag{7}$$

Finally, the parameter estimates are obtained:$A = a$,$B = b$,$C = \sqrt{d^2 + e^2}$, $\phi = \arctan(e/d)$.

2.3 Estimation of $K(m)$

Harness estimation is used to estimate the mean reversion speed (Behowan, 2003).

$$K(m) = -\log\left[\frac{\sum_{i=1}^{N_m}\frac{S(i-1)-T(i-1)}{\sigma_m^2}[T(i)-S(i)]}{\sum_{i=1}^{N_m}\frac{S(i-1)-T(i-1)}{\sigma_m^2}[T(i-1)-S(i-1)]}\right] \tag{8}$$

Where, m =1,2,……,12, represents the mth month of the year, N_m represents the number of days in the month, σ_m represents the fluctuation rate of the temperature in the mth month,$S(i)$ represents the deterministic trend of the ith day, and $T(i)$ represents the true temperature of the ith day.

2.4 Estimation of $\sigma(m)$

By Jing Wang[12], the quadratic variate method is used to estimate $\sigma(m)$:

$$\sigma(m) = \sqrt{\frac{1}{N_m}\sum_{j=0}^{N_m-1}(T_{j+1} - T_j)^2} \tag{9}$$

Where: m =1,2,……,12, represents the nth month of the year,N_m represents the number of days in the month, and T_j represents the temperature on the jth day of the mth month.

2.5 Monte Carlo Method

The Monte Carlo method, also known as random sampling or statistical experimentation method, is based on the law of large numbers as a theoretical basis and fits the real situation by generating a large number of random samples. In this paper, the steps of option pricing using the Monte Carlo method are as follows:

- Given $T(t), S(t), K, \sigma$
- randomly generated $\xi \sim N(0,1)$
- $\Delta T(t) = \Delta S(t) - K(T(t) - S(t)) + \sigma\xi$
- $T(t+1) = T(t) + \Delta T(t)$ （5）
- Repeat steps (1) to (4) Ntimes to obtain$T(n+1), T(n+2), \cdots, T(n+N)$, which in turn gives the price P based on the option pricing formula;
- Repeat step (5) 10,000 times to get $P_1, P_2, \cdots P_{10000}$
- The final option price is $\bar{P} = \frac{1}{10000}\sum_{i=1}^{10000} P_i$

3. RESULT

3.1 Data Sources

In this paper, the daily average temperature from January 1, 1991, to December 31, 2022, in four regions, namely, Shanghai, Quxian, Dongtai, and Dinghai, is used as the modeling data, which is obtained from the National Oceanic and Atmospheric Administration (NOAA) of the United States.

3.2 Parameter Estimation

According to Equation (7), least squares regression was performed on the data using Python software to produce the following parameter estimates:

Table 1: Parameter Estimation Results

Parameter	Estimation	Parameter	Estimation
a	16.5192	A	16.5192
b	0.0001	B	0.0001
d	-4.8867	C	11.6791
e	-10.6076	ϕ	1.1391

From this we can get：

$$S(t) = 16.5192 + 0.0001t + 11.6791\sin(\frac{2\pi}{365}t + 1.1391) \tag{10}$$

The fit of the model is shown below:

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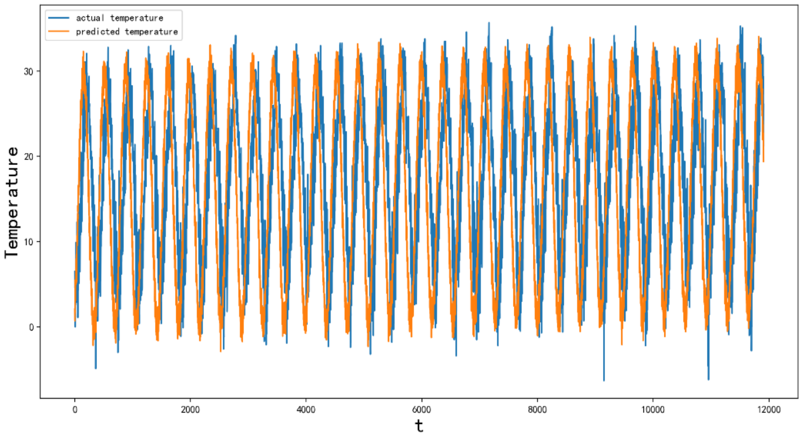

Figure 1: Fitting Curve

From the visualization of the above figure, the fitting effect is relatively good, next the Root Mean Square Error (RMSE) is used to quantitatively observe the fitting effect of the model, which is calculated by the following formula:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(T_i - \hat{T}_i)^2} \tag{11}$$

$n$represents the length of the sequence, T_i represents the real value, and $\widehat{T}_i$ represents the predicted value. Substituting the actual and predicted temperatures, the result is obtained as 0.085, i.e., the average daily prediction error is at $\pm$ 0.085 degrees Celsius.

Then the temperature recovery rate and volatility for each month are obtained from equations (8) and (9). The results are as follows:

Table 2: Mean reversion speed and volatility

Month	σ	K
1	2.3328	0.1289
2	2.6078	0.0686
3	2.7192	0.4490
4	2.5974	0.5250
5	2.2478	0.0707
6	1.8971	0.1592
7	1.6129	0.0863
8	1.3483	0.1714
9	1.4469	0.1277
10	1.6134	0.0274
11	2.4433	0.0694
12	2.5177	0.3467

At this point, all the parameters we need are estimated, and we can use the real-variable O-U model with Monte Carlo methods for temperature forecasting and option pricing.

3.3 Option Pricing

Before introducing the option pricing formula it is necessary to introduce the HeatingDegree-Days (HDD):

$$HDD = \max(18 - X_i, 0) \tag{12}$$

Where: X_i represents the temperature on day i and $N_m$represents the total number of days. The idea behind the formulation of the heat production index is whether the day is overheated or not at the standard of 18 degrees Celsius, and if so, quantify the degree of overheating.

In practice, we are not concerned with the weather conditions on a particular day, but over some time. Therefore an indicator needs to be developed to describe how overheated the weather is over a specific period. A natural idea is to add up the daily heating indices to get a cumulative heating index:

$$HDDs = \sum_{i=1}^{N_m}\max(18 - X_i, 0) \tag{13}$$

Then the pricing formula for a European call option on HDDs temperature is as follows:

$$P_{HDD}(T_1, T_2, K) = e^{-r(T_2 - T_1)} N_p \max\{HDDs(T_1, T_2) - K, 0\} \tag{14}$$

where: T_1, T_2 stands for the start and end time of the contract; $T_2 - T_1$stands for the effective time of this contract; K is the contract execution index, given in advance; r is the risk-free interest rate; and $N_p$is the nominal value of the contract, i.e., how many units of compensation are provided by one unit of the index.

Now assume that the period is from January 1, 2023, to January 31, 2023, then □ $\Delta T = 30$. Let K=100；r=2.6%(five-year Treasury rate); and N_p = 10. We generate 10,000 random paths of temperatures using the Monte Carlo method and then use equation (14) to obtain the corresponding option prices and finally take the average. Combined with the relative error:

$$RE = \frac{|real - prediction|}{real} \times 100\% \tag{15}$$

We get a predicted option price of 1732.33 yuan and an actual option price of 1557.84 yuan, with a relative error of 9.1%.

3.4 Robustness of The Pricing Model

In the above process of option pricing, we applied the Monte Carlo method with stochasticity thus leading to different results for each experiment. This leads to the question, how big is the difference between the results in each case? If the difference is too large it means that the model is not stable enough to be applied. For example, the first time we get an option price of 1000 yuan, and the second time we get an option price of 20 yuan, this error is too large. To explore the stability of the model in the paper, we repeat the pricing process ten times and calculate the HDDs index and option price relative errors separately. Since the actual HDDs index and option price are constant, if the relative error is approximated each time, we can assume that the output of the model does not differ much each time and has application value. The final experimental results show that the relative error of HDDs index is stabilized at 26% and the relative error of option price is stabilized at 9%. This proves the stability of the model and the model can be applied in practice.

Table 3: Relative error in Hdds and option prices

Number of experiments	HDDs	option price
1	26.36%	8.96%
2	26.39%	8.98%
3	26.33%	8.94%
4	26.32%	8.93%
5	26.16%	8.82%
6	26.47%	9.03%
7	26.39%	8.98%
8	26.37%	8.96%
9	26.50%	9.05%
10	26.35%	8.95%

3.5 Pricing of Option Portfolios

Shanghai and Three regions around Shanghai: Dongtai, Quxian, and Dinghai are selected to price the portfolio using the methodology above and calculate the relative error to the actual price. The pricing formula for the portfolio is as follows:

$$P_{HDD}(T_1,T_2,K)=\frac{1}{4}e^{-r(T_2-T_1)}N_p\max\{HDDs(T_1,T_2)-K,0\} \quad (16)$$

Similar to above, let $\Delta T=30$,r=2.6%,N_p=10,K =400. The predicted price was 1,598.12 yuan and the actual price was 1,500.72 yuan, a relative error of 6.5%. The same steps are repeated ten times and the resulting relative error is plotted in the following graph:

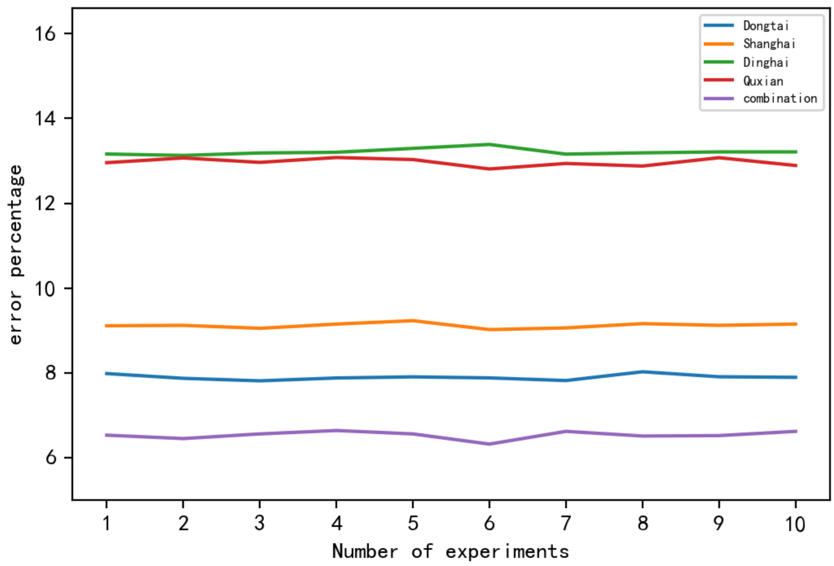

Figure 2: Relative errors in option pricing across cities

4. Conclusion and Discussion

This paper first introduces the establishment and solution of the O-U model and then selects the temperature data of Shanghai from 1991 to 2022 for empirical analysis. Through software modeling, this paper obtains the time-varying O-U model about Shanghai's temperature uses the Monte Carlo method to price the European call option on HDDs index, and obtains the result of 1732.33 yuan, which indicates that the method is feasible. Through several repeated experiments, we find that the relative deviations of the predicted HDDs index, option price, and the respective actual results are at a fixed value of 26% and 9%, respectively, which indicates that the method is stable, does not have excessive deviations, and has application value. By selecting Shanghai, Dongtai, Dinghai, and Quxian to form a portfolio and pricing it, we find that the relative deviation between the theoretical price and the actual price of the portfolio is stable at 6.5%, which is lower than that of any individual city in the portfolio, suggesting that this strategy to improve pricing accuracy is effective. On the basis of this paper, the following research can be conducted: 1. Changing the temperature volatility in the O-U model from changing on a monthly basis to changing on a daily basis may yield more accurate theoretical prices.2. Consider risk hedging strategies in more depth, such as which kinds of weather derivatives need to be purchased in the portfolio of a weather derivative, and how to determine the weights of the purchases.3. Currently, neural-network-based temperature forecasts are selected from classical models, and it is possible to try to introduce more advanced machine learning methods for temperature prediction.

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